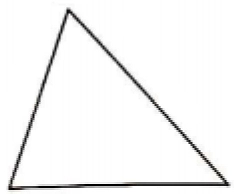
Notes 5.5 Law of Sines

Label the sides and angles of the following triangle. Use **A**, **B**, and **C** to denote angles and **a**, **b**, and **c** to denote sides.



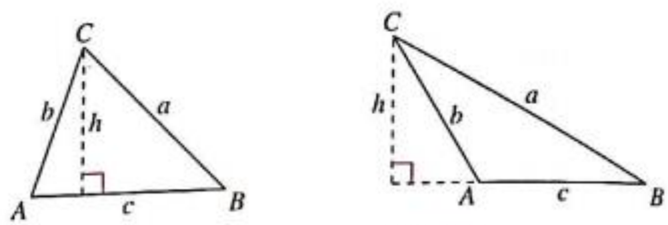
To solve an ________________________, you need to know the measure of at least one side and any two other ___________________of the triangle – either two sides, two angles, or one angle and one side. This breaks down into the following four cases.

1. Two angles and any side (AAS or ASA)
2. Two sides and angle opposite one of them (SSA)
3. Three sides (SSS)
4. Two sides and their included angle (SAS)

Law of Sines

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



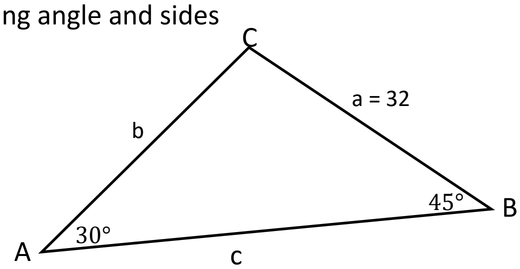
A is acute. A is obtuse.

Example #1 Given Two Angles and One Side - AAS

For triangle ABC, $A = 35°$, $B = 50°$, and a = 16 feet. Find the remaining angles and sides.

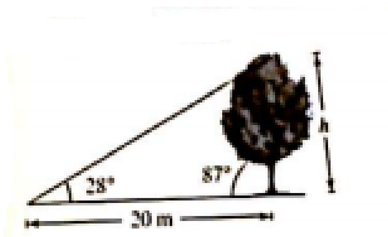
Checkpoint

Find the remaining angle and sides



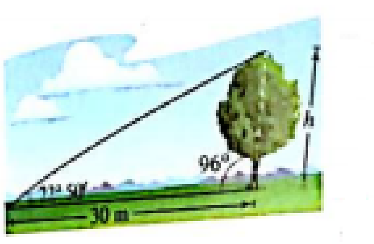
Example 2 Given Two Angles and One Side - ASA

Because of prevailing winds, a tree grew so that it was leaning 3° from the vertical. At a point 20 meters from the tree, the angle of elevation to the top of the tree is 28°. Find the height *h* of the tree.



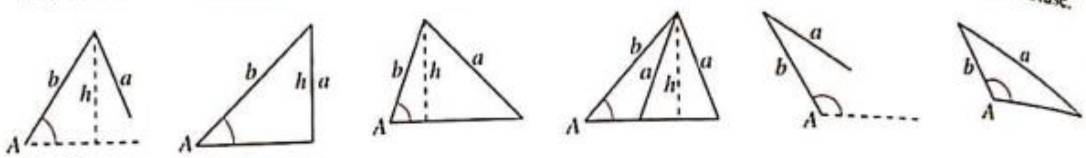
You try:

Find the height of the tree shown below.



The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. $(h = b \sin A)$

	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Triangles possible	None	One	One	Two	None	One

Example #3 Single-Solution Case - SSA

For a triangle with $\boldsymbol{a}$ = 24 inches, $\boldsymbol{b}$ = 15 inches, and $\boldsymbol{A} = 26°$. Find the remaining side and angles.

You try: Given $\boldsymbol{A} = 31°$, $\boldsymbol{a}$ = 12, and $\boldsymbol{b}$ = 5, find the remaining side and angles of the triangle.

Example 4 No-Solution – SSA

Show that that there is no triangle for which $A = 78°$, $\boldsymbol{a}$ = 7, and $\boldsymbol{b}$ = 35.

You try:

Show that there is no triangle for which $A = 85°$, $\boldsymbol{a}$ = 15, and $\boldsymbol{b}$ = 25.

Example 5 Two – Solution Case - SSA

Find two triangles for which $A = 40°$, ***a*** = 12, and ***b*** = 14

You try:

Find two triangles for which $A = 58°$, ***a*** = 4.5, and ***b*** = 5.

Area of an Oblique Triangle

Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,

$$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B.$$

Example 6: Finding the Area of a Triangular Lot

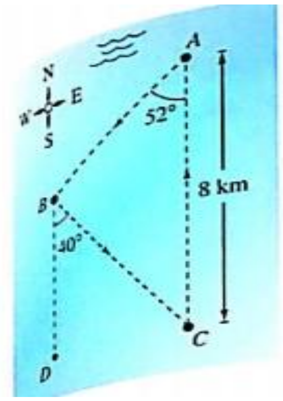
Find the area of a triangular lot containing side lengths that measure 84 yards and 55 yards and form an angle of $115°$.

You try:

Find the area of a triangular lot having two sides of lengths 24 inches and 18 inches and an included angle of $80°$.

Example 7: An Application of the Law of Sines

a. The course for a boat races starts at point *A* and proceeds in the direction S $52°$ W to point *B*, then in the direction S $40°$ E to point *C*, and finally back to point *A*. Point *C* lies 8 kilometers directly south of point *A*, Approximate the total distance of the race course. See figure below



b. A boat travels from point *A* to point *B* at a bearing of N $82°$ W. The boat then travels to point *C* at a bearing of S $36°$ E. Point *C* is 15 miles due south of point *A*. How many total miles does the boat travel?

You try: On a small lake, you swim from point *A* to point *B* at a bearing of N $28°$ E, then to point *C* at a bearing of N $58°$ W, and finally back to point *A*, as shown in the figure below. Point *C* lies 800 meters directly north of point *A*. Approximate the total distance that you swim.